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Glueball Masses for the Deformed Conifold Theory

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Abstract

We obtain the spectrum of glueball masses for the $\mathcal{N} = 1$ non-conformal cascade theory whose supergravity dual was recently constructed by Klebanov and Strassler. The glueball masses are calculated by solving the supergravity equations of motion for the dilaton and the two-form in the deformed conifold background.

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1 Introduction

The original AdS/CFT correspondence [1]-[3] was generalized in [4]-[8] for branes at conical singularities (see also [9]-[13] for related work). If instead of locating D3-branes in a flat transversal space they are placed at the vertex of a six dimensional cone with base a five dimensional Einstein manifold X_5, one is lead to conjecture that type IIB string theory on $AdS_5 \times X_5$ is dual to the low energy limit of the worldvolume theory on the D3-branes at the singularity. In particular, a set of N D3-branes at the conifold singularity results on a $\mathcal{N}=1$ superconformal field theory with $SU(N) \times SU(N)$ gauge group [7]. Conformal invariance can be broken using fractional D3-branes which are allowed to appear in certain singular spaces [14]-[17]. Fractional D3-branes arise from D5-branes wrapped at the collapsed two-cycles of the singularity. In particular, the addition of M fractional branes at the singular point of the conifold modifies the gauge group of the field theory to $SU(N+M) \times SU(N)$. This theory was first investigated in [17] to leading order in M/N. This solution was then completed to all orders in [18]. However, the result found by the authors of [18] contains an infrared singularity. In [19], Klebanov and Strassler constructed a non singular solution valid from the ultraviolet to the infrared. They showed that the theory undergoes a series of Seiberg dualitites as the gauge group is sucesively broken. Far in the infrared the D3-branes disappear, and the theory is $\mathcal{N}=1$ $SU(M)$ with no matter; this theory exhibits confinement, domain walls and screening. Using the supergravity description of gauge theories, as in [20]-[22], we will in the present letter find glueball masses for $\mathcal{N}=1$ $SU(M)$ Yang-Mills. In section 2 we will review some of the aspects of the solution constructed in [19], emphasizing those relevant to our calculation. In section 3 we analyze the equations of motion of type IIB supergravity, and their final form after dimensional reduction on a sphere. The equations for the type IIB dilaton and the two-form fields are numerically solved in section 3 to determine the mass spectra of the corresponding glueball modes. Finally, in section 4, we present some conclusions and comment on future perspectives[1].

[1]After completion of this paper, reference [23] appeared, which partially overlaps with the present work.

2 Review of Klebanov-Strassler's Solution

The solution recently found by Klebanov and Strassler arises from the study of D3-branes at a singular space. Originally, the authors of [7] studied the conformal field theory on D3-branes at a Calabi-Yau singularity dual to type IIB on a $AdS_5 \times T^{11}$ background. The corresponding gauge theory is $\mathcal{N} = 1$ supersymmetric with $SU(N) \times SU(N)$ gauge group, and matter content A_i,B_i where $i = 1, 2$. The chiral fields transform as $(\mathbf{N}, \bar{\mathbf{N}})$ and $(\bar{\mathbf{N}}, \mathbf{N})$ respectively. The superpotential is $W = \lambda\epsilon^{ij}\epsilon_{kl}\mathrm{Tr}\ A_i B_k A_j B_l$. In the presence of M fractional branes the superpotential and matter content are the same but the gauge group changes to $SU(N + N) \times SU(N)$. The chiral superfields are now in the representation $(\mathbf{N} + \mathbf{M}, \bar{\mathbf{N}})$. The supergravity equations corresponding to this situation were solved, to leading order in M/N, in [17], where the relative gauge coupling $g_1^{-2} - g_2^{-2}$ was shown to run logarithmically. This approximation was completed to all orders in [18]. In this solution a logarithmic harmonic function warps the conifold,

$$ds^2 = \frac{r^2}{L^2\sqrt{\ln(r/r_s)}}dx_n dx_n + \frac{L^2\sqrt{\ln(r/r_s)}}{r^2}dr^2 + L^2\sqrt{\ln(r/r_s)}ds^2_{T^{11}}. \tag{2.1}$$

These fractional D3-branes at the singularity are D5-branes wrapped over the collapsed S^2 of T^{11}. D5-branes are sources for the magnetic R-R three-form flux through the S^3 cycle of T^{11} and thus adding M fractional D3-branes implies that the supergravity dual of this field theory will involve M units of three-form flux,

$$\int_{S^3} F_3 = M, \tag{2.2}$$

in addition to N units of five-form flux coming from the D3-branes,

$$\int_{T^{11}} F_5 = N. \tag{2.3}$$

This non vanishing three-form is responsible of the conformal symmetry breaking. As a consequence of this flux, the two-form B_2 is no longer constant; it develops a radial dependence [17],

$$\int_{S^2} B_2 \sim M e^{\phi} \ln(r/r_s), \tag{2.4}$$

while the dilaton remains constant.

The supergravity solution obtained in [17] was completed in [18] taking into account the back reaction of $H_3 = dB_2$ and F_3 on other fields. This exact solution exhibits an effect which was hidden to leading order in M: since $F_5 = dC_4 + B_2 \wedge F_3$, F_5 also acquires a radial dependence,

$$F_5 = \mathcal{F}_5 + \star\mathcal{F}_5, \tag{2.5}$$

where

$$\mathcal{F}_5 = \mathcal{K}(r)\text{vol}\ (T^{11}) = (N + ag_sM^2\ln(r/r_0))\text{vol}\ (T^{11}), \tag{2.6}$$

with a a constant of order unity. But there is a novelty arising from this solution: the five-form flux present at the ultraviolet scale $r = r_0$ may disappear once we reach the scale $r = \tilde{r}$, where $\mathcal{K}(\tilde{r}) = 0$. This phenomenon can be related to the fact that the flux $\int_{S^2} B_2$ is not a periodic variable in the supergravity solution, because as this flux goes through a period, $\mathcal{K}(r) \to \mathcal{K}(r) - M$, so that the five-form flux is decreased by M units. This decrease represents a renormalization group cascade, that was identified in [19] as a form of Seiberg duality [24].

However, the metric (2.1), representing the logarithmic renormalization group cascade, contains a naked singularity at $r = r_s$, which is the point where the harmonic function vanishes, $h(r_s) = 0$. From a physical point of view the singularity represents the end point of the cascade as the theory flows to the infrared, because negative values of N are unphysical. And it is this singularity in the metric (2.1), that demands a modification, at least in the infrared, of the solution. This non singular correction was found in [19], where it was argued that the conifold should be replaced by a deformed conifold,

$$\sum_{i=1}^{4} z_i^2 = -2\det_{ij} z_{ij} = \epsilon^2, \tag{2.7}$$

with the singularity removed through the blow up of the S^3 in T^{11}. A simple argument in favor of this suggestion comes from the origin of this singularity; it can be related to the divergent energy of the F_3 field. As there are M units of flux of F_3 through the S^3 in T^{11}, when S^3 shrinks to zero size F_3 diverges. However, if S^3 is kept of finite size, as in the deformed conifold, there is no need for F_3 to diverge.

A deeper argument comes from a detailed field theory analysis, which shows that the spacetime geometry should be modified by the strong dynamics of the infrared limit of

the field theory. The authors of [19] showed how the $U(1)$ ($\mathbf{Z}_{2M}$, to be more precise) R-symmetry is broken to a $\mathbf{Z}_2$ symmetry. This is indeed the symmetry left unbroken on the deformed conifold (2.7), $z_k \to -z_k$, instead of the original $U(1)$ $z_k \to e^{i\alpha} z_k$. The metric was then shown to be of the form

$$ds_{10}^2 = h^{-1/2}(\tau) dx_n dx_n + h^{1/2}(\tau) ds_6^2, \tag{2.8}$$

with ds_6^2 the metric of the deformed conifold. In the basis $\{\tau, g^{i=1,\cdots,5}(\psi, \theta_1, \theta_2, \phi_1, \phi_2)\}$ of reference [25] this metric becomes diagonal,

$$\begin{aligned} ds_6^2 = & \frac{1}{2}\epsilon^{4/3} K(\tau) \Big[\frac{1}{3K^3(\tau)} [d\tau^2 + (g^5)^2] + \cosh^2\left(\frac{\tau}{2}\right) [(g^3)^2 + (g^4)^2] \\ & + \sinh^2\left(\frac{\tau}{2}\right) [(g^1)^2 + (g^2)^2] \Big], \end{aligned} \tag{2.9}$$

where

$$K(\tau) = \frac{(\sinh(2\tau) - 2\tau)^{1/3}}{2^{1/3}\sinh(\tau)}. \tag{2.10}$$

The harmonic function in (2.8) is given by the integral expression

$$h(\tau) = \alpha \frac{2^{2/3}}{4} \int_\tau^\infty dx \frac{x \coth x - 1}{\sinh^2 x} (\sinh(2x) - 2x)^{1/3}, \tag{2.11}$$

which cannot be evaluated in terms of elementary or special functions. The constant α is $\alpha \sim (g_s M)^2$.

The solution contains a five-form and three-form flux. $F_5 = \mathcal{F}_5 + \star\mathcal{F}_5$ is given by;

$$\mathcal{F}_5 = g_s M^2 l(\tau) g^1 \wedge g^2 \wedge g^3 \wedge g^4 \wedge g^5, \tag{2.12}$$

and

$$\star\mathcal{F}_5 = g_s M^2 \frac{2\, l(\tau)}{15\, K^2(\tau) h^2(\tau) \sinh^2(\tau) \epsilon^{8/3}} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge d\tau, \tag{2.13}$$

while the three-form is

$$G_3 = F_3 + \frac{i}{g_s} H_3 \tag{2.14}$$

$$\begin{aligned} = & M\big\{ g^5 \wedge g^3 \wedge g^4 + d[F(\tau)(g^1 \wedge g^3 + g^2 \wedge g^4)] \\ & + i d[f(\tau) g^1 \wedge g^2 + k(\tau) g^3 \wedge g^4] \big\}, \end{aligned} \tag{2.15}$$

with the functions

$$
\begin{aligned}
l(\tau) &= \frac{\tau\coth\tau - 1}{4\sinh^2\tau}(\sinh 2\tau - 2\tau), \\
f(\tau) &= \frac{\tau\coth\tau - 1}{2\sinh\tau}(\cosh\tau - 1), \\
k(\tau) &= \frac{\tau\coth\tau - 1}{2\sinh\tau}(\cosh\tau + 1), \\
F(\tau) &= \frac{\sinh\tau - \tau}{2\sinh\tau}.
\end{aligned}
\tag{2.16}
$$

In the far infrared, through the chain of Seiberg dualities that drop the size of the gauge group factors by M units, the D3-brane goes to zero and only the M fractional D3-branes remain. Thus, at the bottom of the cascade we are left with a pure $\mathcal{N} = 1$ Yang-Mills theory with M isolated vacua. In this letter we will use the above supergravity description of this theory, proposed by Klebanov and Strassler, to study the glueball mass spectra of this theory by solving numerically the equations of motion describing the propagation of supergravity fields along the worldvolume of the branes.

3 Supergravity Equations

In this section we will write down the dimensional reduction of the linearized equations of motion of type IIB supergravity in the background described in section 2. The bosonic sector of the type IIB supergravity multiplet contains a graviton $g_{\hat{\mu}\hat{\nu}}$, a dilaton Φ, a zero-form R-R field C, two tensors –the NS-NS and R-R fields $B_{\hat{\mu}\hat{\nu}}$ and $C_{\hat{\mu}\hat{\nu}}$– and a R-R four-form $C_{\hat{\sigma}\hat{\lambda}\hat{\tau}\hat{\nu}}$. In the present notation carets denote indices that run over ten dimensions, greek indices denote four dimensional space and latin indices run over the internal space. Expanding the type IIB equations of motion [26] in background (dotted

fields) and fluctuations we get,

$$\begin{aligned}
D^{\hat{\mu}}\partial_{\hat{\mu}}\Phi &= \frac{\kappa^2}{24}\dot{G}_{\hat{\rho}\hat{\sigma}\hat{\tau}}G^{\hat{\rho}\hat{\sigma}\hat{\tau}}, && (3.1)\\
D^{\hat{\mu}}G_{\hat{\mu}\hat{\rho}\hat{\sigma}} &= -\frac{2i\kappa}{3}[F_{\hat{\rho}\hat{\sigma}\hat{\lambda}\hat{\tau}\hat{\nu}}\dot{G}^{\hat{\lambda}\hat{\tau}\hat{\nu}} + \dot{F}_{\hat{\rho}\hat{\sigma}\hat{\lambda}\hat{\tau}\hat{\nu}}G^{\hat{\lambda}\hat{\tau}\hat{\nu}}] + \partial^{\hat{\mu}}\Phi\dot{G}_{\hat{\mu}\hat{\rho}\hat{\sigma}}, && (3.2)\\
R_{\hat{\mu}\hat{\nu}} &= \frac{\kappa^2}{6}F_{\hat{\mu}\hat{\rho}\hat{\sigma}\hat{\lambda}\hat{\tau}}\dot{F}_{\nu}{}^{\hat{\rho}\hat{\sigma}\hat{\lambda}\hat{\tau}} + \frac{\kappa^2}{8}[\mathrm{Re}(\dot{G}_{\hat{\mu}}{}^{\hat{\rho}\hat{\sigma}}G_{\hat{\nu}\hat{\rho}\hat{\sigma}}) - \frac{1}{6}g_{\hat{\mu}\hat{\nu}}\dot{G}^{\hat{\sigma}\hat{\lambda}\hat{\tau}}\dot{G}_{\hat{\sigma}\hat{\lambda}\hat{\tau}} \\
&- \frac{1}{6}\dot{g}_{\hat{\mu}\hat{\nu}}\dot{G}^{\hat{\sigma}\hat{\lambda}\hat{\tau}}G_{\hat{\sigma}\hat{\lambda}\hat{\tau}}], && (3.3)\\
F_{\hat{\mu}_1\hat{\mu}_2\hat{\mu}_3\hat{\mu}_4\hat{\mu}_5} &= \frac{1}{5\,!}\varepsilon_{\hat{\mu}_1\hat{\mu}_2\hat{\mu}_3\hat{\mu}_4\hat{\mu}_5\ldots\hat{\mu}_{10}}F^{\hat{\mu}_6\ldots.\hat{\mu}_{10}} && (3.4)
\end{aligned}$$

The presence of a non zero three-form in the background implies the coupling of the dilaton, metric and two form. In general this should require solving the system of coupled equations. However, these equations can be simplified by expanding the excitations in spherical harmonics [27]. Formally, one should expand in harmonics over the exact background. Nevertheless, since we are interested in calculating glueball masses, *i.e.*, in confining effects, and these occur near the bottom of the cascade where the the S^2 shrinks to zero but the S^3 does not we can expand in Kaluza-Klein modes over the S^3. The expansion for the dilaton is,

$$\Phi(x,y) = \sum \phi(x)^k Y(y)^k, \tag{3.5}$$

while for the two-form

$$\begin{aligned}
A_{\mu\nu}(x,y) &= \sum a^{I_1}_{\mu\nu}(x)Y^{I_1}(y),\\
A_{\mu m}(x,y) &= \sum [a^{I_5}_{\mu}(x)Y^{I_5}_m(y) + a^{I_1}_{\mu}(x)D_m Y^{I_1}(y)],\\
A_{mn}(x,y) &= \sum [a^{I_{10}}(x)Y^{I_{10}}_{[mn]}(y) + a^{I_5}(x)D_{[m}Y^{I_5}_{n]}(y)], && (3.6)
\end{aligned}$$

where $x = x^\mu$ are four dimensional worldvolume coordinates are $y = x^i$ are coordinates over the three sphere. The Lorentz type gauges $D^m A_{mn} = 0$, $D^m A_{m\mu} = 0$ can be used in order to fix $a^{I_1}_\mu = a^{I_5} = 0$ in (3.6).

First, we will consider the equation (3.1) for the dilaton which only requires scalar spherical harmonics. Substituting the expansion (3.5) into the field equation for the dilaton we see that the s-wave of the kinetic side becomes $D^\mu\partial_\mu\phi(x)\,Y(y)$, where we have omitted the $k = 0$ index. However, the three-form of the Klebanov-Strassler solution [19]

lives only on the internal coordinates so its spherical harmonic expansion involves $Y_{[mn]}$ but no scalar harmonics. Thus, the s-wave equation for the dilaton becomes $D^{\mu}\partial_{\mu}\phi(x) = 0$ or, explicitly,

$$\frac{1}{\sqrt{g}}\partial_{\mu}[\sqrt{g}\partial_{\nu}\phi g^{\mu\nu}] = 0. \tag{3.7}$$

In order to study the propagation of the supergravity two-form along the worldvolume of the branes we should consider the $D^{\hat{\alpha}}G_{\hat{\alpha}\mu\nu}$ part of equation (3.2). Using

$$C_{\mu mnp} = \sum \phi_{\mu}^{I_5}(x)\varepsilon_{mnp}{}^{qr}D_q Y_r^{I_5}(y) \tag{3.8}$$

together with (3.5) and (3.6) the s-wave equation of motion for the two-form in the worldvolume is

$$\frac{3}{\sqrt{g}}g_{\mu\mu'}g_{\nu\nu'}\partial_{\alpha}[\sqrt{g}\partial_{[\alpha'}a_{\mu''\nu'']}g^{\alpha\alpha'}g^{\mu''\mu'}g^{\nu''\nu'}] = -\frac{2i\kappa}{3}\dot{F}_{\mu\nu\rho\sigma\tau}\partial^{[\rho}a^{\sigma\tau]}, \tag{3.9}$$

where [ ] denotes antisymmetrization with strength one, and $a_{\mu\nu}$ is the complexified two-form.

4 Glueball Mass Spectra

In this section we will study the discrete spectrum of glueball masses arising from propagation in (2.8) of the type IIB dilaton field, and the complex antisymmetric field $a_{\mu\nu}$. The $SU(N+M)\times SU(N)$ conifold theory has a $U(1)_R$ global symmetry [18]. In the present case, the deformation of the conifold breaks the $U(1)_R$ to $\mathbf{Z}_2$. Therefore, one expects a massless glueball in this theory. However, this massless glueball will couple to some combination of supergravity fields[2], and thus we do not expect to find it in our analysis of the glueball spectra.

In [2, 3] the correspondence between the $AdS_5\times S^5$ background and primary chiral fields correlators was made explicit. This correspondence should, in principle, be modified for the deformed conifold background since the space is not asymptotically AdS anymore. However, as argued in [19], only operators with $\Delta < \frac{3}{2}M$ can propagate all the way to $\tau = 0$. For this type of operators one expects that the correspondence should not be

[2]We thank I. Klebanov for comments on this point.

greatly modified. Thus, one expects that the dilaton will couple to the dimension four operator Tr $F_{\mu\nu}F^{\mu\nu}$, so that the spin zero glueball masses, which can be derived from the two point function of this operator, will arise from propagation of the dilaton. Similarly, the supergravity two-form should couple to a dimension six operator, *i.e.*, to the spin one glueball.

In order to obtain a pure glue gauge theory we should take the $g_s M \to 0$ limit. In this limit the curvature of the space is large and we are outside of the region where supergravity has small corrections. This situation is not new when dealing with string duals of pure glue theories. Thus, in the spirit of [21, 22] we proceed with the calculation and expect the corrections to masses to be small. With this caveat let us now consider the equation for the dilaton. Expanding the dilaton field in plane waves, $\phi(\tau, x) = f(\tau)e^{ik.x}$, so that a mode of momentum k has a mass $m^2 = -k^2$ and using the metric (2.8) the dilaton equation (3.7) becomes

$$3.2^{1/3}\frac{d}{d\tau}\left[(\sinh(2\tau) - 2\tau)^{2/3}\frac{df}{d\tau}\right] - (k^2\epsilon^{4/3})\sinh^2(\tau)h(\tau)f = 0. \tag{4.1}$$

As a boundary condition we will require that near the origin the function f must be smooth, so that $df/d\tau = 0$ at $\tau = 0$. The asymptotics of $f(\tau)$ as $\tau \to \infty$ is obtained by demanding normalizability of the states. Since for large τ,

$$\sqrt{g} = 2^{-5}3^{-1}\epsilon^4\sinh^2(\tau)h^{1/2}(\tau) \sim \tau e^{2/3\tau} \tag{4.2}$$

convergence of the integral $\int\sqrt{g}\,|\Phi|^2$ signals an exponential behaviour of the solution near infinity, $f \sim e^{n\tau}$, as can be suspected from direct inspection of (4.1). Changing variables to $f(\tau) = \psi(\tau)e^{n\tau}$, the wave equation (4.1) becomes

$$\begin{aligned}&6[\sinh(2\tau) - 2\tau]^{2/3}\,\psi'' + [6n2^{1/3}(\sinh(2\tau) - 2\tau)^{2/3}\\&+ 2^{7/3}(\cosh(2\tau) - 1)(\sinh(2\tau) - 2\tau)^{-1/3}]\,\psi' + [3n^2 2^{1/3}(\sinh(2\tau) - 2\tau)^{2/3}\\&+ 2^{7/3}n(\cosh(2\tau) - 1)(\sinh(2\tau) - 2\tau)^{-1/3} - (k^2\epsilon^{4/3})\sinh^2(\tau)h(\tau)]\,\psi = 0,\end{aligned} \tag{4.3}$$

This equation can be solved for large values of τ by $\psi(\tau) = c_1 e^{-n\tau} + c_2 e^{-\frac{1}{3}(3n+4)\tau}$ which implies $f(\tau) = c_1 + c_2 e^{-4/3\tau}$. Normalizability requires $c_1 = 0$ and we can fix $c_2 = 1$. Thus, at infinity $f \sim e^{-4/3\tau}$. Equation (4.1) becomes then an eigenvalue problem; we want to find the values of k for which the equation has solutions matching the boundary conditions at 0 and ∞. We have done this numerically using a shooting technique [21]:

boundary conditions at infinity are taken as initial conditions and k is adjusted so that when numerically integrating (4.1) f is also smooth at $\tau = 0$. We should note that, numerically, infinity is taken as some large value of τ such that $\tau \gg k^2$. On the other hand, since the boundary condition requires that f goes exponentially fast to zero at infinity, there is a numerical bound that does not allow us to push the numerical infinity as far as we want. This explains why we are restricted to obtain only the first two eigenvalues. The glueball masses thus obtained are shown in Table 1. They are measured in units of $\epsilon^{4/3}$ which, as explained in [19], sets the four dimensional mass scale of the field theory.

State	**(Mass)^2**
0^{++}	9.78
0^{++*}	33.17

Table 1: Mass (squared) of the spin zero glueball and its first excited state obtained from supergravity.

It is interesting to note that by varying the origin of integration one can see confining effects occurring in a small region close to the bottom of the cascade. Past this small region the spectrum becomes continuous signaling a conformal behaviour.

Next we will study the equation of motion for the complexified two-form $a_{\mu\nu}$. Using again the metric (2.8) and the five-form flux through the worldvolume, (2.13), equation (3.9) becomes

$$\begin{aligned} &3.2^{1/3}\frac{d}{d\tau}\left[h(\tau)(\sinh(2\tau)-2\tau)^{2/3}\frac{df_{\mu\nu}}{d\tau}\right]-(k^2\epsilon^{4/3})\sinh^2(\tau)h^2(\tau)f_{\mu\nu} \\ &\qquad = -\frac{8i\kappa}{15}g_s M^2 l(\tau)\epsilon^{-2}\left[\varepsilon_{\mu\nu\tau\alpha\beta}\epsilon^{-2/3}\frac{df_{\alpha\beta}}{d\tau}+ik\frac{\epsilon^{2/3}h(\tau)}{6K^2(\tau)}\varepsilon_{\mu\nu x\alpha\beta}f_{\alpha\beta}\right], \end{aligned} \tag{4.4}$$

where we have impossed solutions to be of the form $a_{\mu\nu}(\tau,x) = f_{\mu\nu}(\tau)e^{ik.x}$, and summation is implied over the α and β indices. We should note that as a consequence of the non constant five-form background of the solution of Klebanov and Strassler, the second order differential operator (3.9) can not be factorized into two first order operators, as in [27]. Thus, we should take care explicitly of the derivatives in the right hand side of (3.9). This requires the complex decomposition $f_{\mu\nu}(\tau) = b_{\mu\nu}(\tau) + ic_{\mu\nu}(\tau)$. Impossing the gauge fixing condition $a_{\mu\tau} = 0$, so that the two-form has no components along the radial direction,

equation (4.4) becomes a system of coupled equations. Similarly as in the dilaton case, the glueball masses can be obtained by numerically solving this system. The values are shown in Table 2.

State	(Mass)^2
1^{--}	14.05
1^{--*}	42.90

Table 2: Mass (squared) of the spin one glueball and its first excited state obtained from supergravity.

5 Discussion

In this paper we have computed the dilaton and two-form excitations in the deformed conifold background recently constructed by Klebanov and Strassler [19]. Via the correspondence between supergravity and field theory, this corresponds to determining masses for glueballs in the dual effective field theory. Unlike the $AdS_5 \times S^5$ case, the Klebanov-Strassler background contains non trivial three-form and five-form fluxes. Far in the infrared the three-form flux prevents the three-cycle of the base from collapsing. We obtained the linearized type IIB equations by doing a Kaluza-Klein decomposition on this S^3. We solved the resulting eigenvalue problem by numerically integrating these equations. This method is exact and allows us to find the first two excited states for the dilaton and two-form fluctuations. A WKB approximation could also be used to solve the equations. This approach was used in [28]-[30] to determine glueball masses for several finite temperature supergravity models. A comparison of WKB results with those of [21], where a numerical method was used, shows that agreement increases for excited states. This is to be expected since the WKB approximation improves for large values of the masses. This might be at the root of the small discrepancy, when comparing ratios of eigenvalues, between our results for the spin zero glueball and those in [23], where the WKB approximation was used. Due to the fact that the equations involve extremely divergent functions our numerical method is not able to find higher excited states and we cannot compare with [23] for states where the WKB approximation is more reliable. It would be interesting to compare with lattice results but we are not aware of any lattice computation for supersymmetric glueballs.

The present computations can be generalized to higher spin glueballs. It would also be interesting to study mixed supergravity states; one would then expect to find a massless glueball as a consequence of the breaking of the $U(1)_R$ symmetry.

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